

On The Transport Theory of Classical Plasma in Rindler Space

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We have obtained the Vlasov equation and Boltzmann kinetic equation using the classical Hamilton equation and Poisson bracket with Rindler Hamiltonian. We treat the whole Universe as a statistical system with galaxies as the point particle constituents in large scale structure. Since the collisions of galaxies are very rare phenomena, we assume that the gas with the constituents as point galaxies satisfy Vlasov equation. Considering the astrophysical catastrophic event, e.g., the generation of gravity waves by the collisions of black holes in one of the galaxies, and further assuming that when such a wave passes through the gas causes a kind of asymmetry in mass distribution. We may call it a kind of polarization (of course there is no mass and anti-mass). This polarization of mass distribution will further gives rise to gravitational permittivity or dielectric constant. We have shown that the low frequency gravity waves will be absorbed, whereas the high frequency part will pass through the gas of point galaxies. It is further noticed that the region in space with extremely high gravitational field is transparent to gravity waves. In the other part of this work, using the Boltzmann equation and replacing the collision term by the relaxation time approximation and further assuming a small deviation from the equilibrium configuration of the stellar / galactic plasma in Rindler space, we have obtained the kinetic coefficients. For the first time we have derived an expression for the coefficient of gravitational flow. It has further been shown that in presence of strong gravitational field all the kinetic coefficients become vanishingly small, so the matter behaves like an ideal fluid, which may happen near the event horizon of black holes.

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1. INTRODUCTION

Exactly like the Lorentz transformations of space time coordinates in the inertial system of frames, the Rindler coordinate transformations are for the uniformly accelerated frame of references [1, 2]. The space-time geometrical structure is called the Rindler space. From the references [3–10], it can very easily be shown that the Rindler coordinate transformations are given by:

$$\begin{aligned} ct &= \left(\frac{c^2}{\alpha} + x' \right) \sinh \left(\frac{\alpha t'}{c} \right) \quad \text{and} \\ x &= \left(\frac{c^2}{\alpha} + x' \right) \cosh \left(\frac{\alpha t'}{c} \right) \end{aligned} \quad (1)$$

Hence it is a matter of simple algebra to prove that the inverse transformations are given by:

$$ct' = \frac{c^2}{2\alpha} \ln \left(\frac{x + ct}{x - ct} \right) \quad \text{and} \quad x' = (x^2 - (ct)^2)^{1/2} - \frac{c^2}{\alpha} \quad (2)$$

Here α indicates the uniform acceleration of the frame along positive x -direction. Hence it can very easily be shown from eqns.(1) and (2) that the square of the four-line element changes from

$$\begin{aligned} ds^2 &= d(ct)^2 - dx^2 - dy^2 - dz^2 \quad \text{to} \\ ds^2 &= \left(1 + \frac{\alpha x'}{c^2} \right)^2 d(ct')^2 - dx'^2 - dy'^2 - dz'^2 \end{aligned} \quad (3)$$

where the former line element is in the Minkowski space, whereas the later one is in Rindler space. Hence the metric in the Rindler space can be written as

$$g^{\mu\nu} = \text{diag} \left(\left(1 + \frac{\alpha x}{c^2} \right), -1, -1, -1 \right) \quad (4)$$

whereas in the Minkowski space-time we have the usual form

$$g^{\mu\nu} = \text{diag}(+1, -1, -1, -1) \quad (5)$$

It is therefore quite obvious that the Rindler space is also flat. The only difference from the Minkowski space is that the frame of the observer is moving with uniform acceleration. In 1 + 1-dimension, the Rindler metric is given by

$$g^{\mu\nu} = \text{diag} \left(\left(1 + \frac{\alpha x}{c^2} \right), -1 \right)$$

It has been noticed from the literature survey that the principle of equivalence plays an important role in obtaining the Rindler coordinates in the uniformly accelerated frame of reference. According to this principle an accelerated frame in absence of gravity is equivalent to a frame at rest in presence of gravity. Therefore in the present scenario, α may also be treated to be the strength of constant gravitational field for a frame at rest.

Now from the relativistic dynamics of special theory of relativity, the action integral is given by [1]

$$S = -\alpha_0 c \int_a^b ds \equiv \int_a^b L dt \quad (6)$$

where $\alpha_0 = -m_0 c$ and m_0 is the rest mass of the particle and c is the speed of light in vacuum [1]. The Lagrangian of the particle may be written as

$$L = -m_0 c^2 \left[\left(1 + \frac{\alpha x}{c^2} \right)^2 - \frac{v^2}{c^2} \right]^{1/2} \quad (7)$$

where v is the component of three velocity along positive x -direction. Hence the component of three momentum of the particle along positive x -direction is given by

$$p = \frac{\partial L}{\partial v}, \quad \text{or} \quad (8)$$

$$p = \frac{m_0 v}{\left[\left(1 + \frac{\alpha x}{c^2} \right)^2 - \frac{v^2}{c^2} \right]^{1/2}} \quad (9)$$

Then from the definition, the Hamiltonian of the particle may be written as

$$H = pv - L \quad \text{or} \quad (10)$$

$$H = m_0 c^2 \left(1 + \frac{\alpha x}{c^2} \right) \left(1 + \frac{p^2}{m_0^2 c^2} \right)^{1/2} \quad (11)$$

Hence it can very easily be shown that in the non-relativistic approximation, the Hamiltonian is given by

$$H = \left(1 + \frac{\alpha x}{c^2} \right) \left(\frac{p^2}{2m_0} + m_0 c^2 \right) \quad (11a)$$

In this study our intention is to show that the gravity waves and the electromagnetic waves behave differently in presence of strong gravitational field. For the gravity waves, the refractive index will be real and constant, therefore a plane gravity wave will travel in straight line without any deviation near the event horizon. Whereas it is well established that there will be bending of electromagnetic waves near strong gravitational field. This bending can also be explained by the variation of refractive index. One can show very easily (see [1, 11]) that the refractive index increases towards the increasing gravitational field. It can also be shown that for the electromagnetic waves, gravity behaves like a refracting medium with changing refractive index as the gravity changes in the free space. In the second part of this investigation we have shown that in presence of strong gravitational field all the kinetic coefficients become almost zero. Therefore a stellar plasma in presence of strong gravitational field will behave like an ideal fluid.

The manuscript is organized in the following manner: In the next section considering the whole universe to be a statistical system, and further assuming that the galaxies are point objects in large scale structure, we have developed a formalism to obtain the Vlasov equation satisfied by this point objects. With a harmonic type perturbation by strong gravity waves, we have obtained the gravitational permittivity of the medium and studied its variation with frequency and also with the strength of gravitational field. In section 3 we have obtained the kinetic coefficients for a stellar plasma using the Boltzmann kinetic equation in Rindler space. To the best of our knowledge such studies have not been done before.

2. VLASOV EQUATION IN RINDLER SPACE

We assume that the whole universe is a statistical system and in the large scale structure the galaxies are considered to be the point particles, the constituents of this statistical system. Since the collision of galaxies is a very rare phenomena, we may assume that these point structure galaxies satisfy Vlasov equation. We further assume that the gas is in an average uniform gravitational field α . The space time geometry we have considered is the so called Rindler space.

To obtain the Vlasov equation, we start with the well known Hamilton equation, given by

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\} \quad (12)$$

where distribution function $f(x, p, t)$ is a function of space, time and momentum coordinates of the particle. Now using the definition of Poisson bracket, we have

$$\{f, H\} = \{f, H\}_{x,p} = \frac{\partial f}{\partial x} \frac{\partial H}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial H}{\partial x} \quad (13)$$

Therefore for the Rindler Hamiltonian $H(x, p)$, given by eqn.(11), we have for the collision-less case ($df/dt = 0$),

$$\frac{\partial f}{\partial t} - v\beta(x) \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{-1/2} \frac{\partial f}{\partial x} - m_0 \alpha \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2} \frac{\partial f}{\partial p} = 0 \quad (14)$$

This is the Vlasov equation in Rindler space satisfied by the galaxies assumed to be point particles in large scale structure. We further assume that the average mass of the galaxies is m_0 (see [12] for the electromagnetic case). Let us now assume some kind of astrophysical catastrophic events, e.g., generation of very high energy gravity waves by the collision of black holes in one of the point galaxies. The physical picture is that the gravity waves are coming out from one of these point particles and falls on the collision-less plasma of galaxies. Although there are many harmonics present in the gravity waves, we start with a monochromatic gravity waves and later investigate the interaction of very low and very high frequency gravity waves on such collision-less plasma. We assume that such high intensity gravity waves falling on this collision-less plasma causes some kind of polarization of mass distribution. This is of course not exactly like the electrostatic case. It is the asymmetry of mass distribution of plasma particles caused by the incident gravity waves. This polarization effect may be represented by the deviation of the distribution function from its equilibrium configuration. We assume that the deviation is low enough, which we express mathematically as the first order deviation from the equilibrium distribution, denoted by

$$f(x, p, t) = f_0(p) + \delta f(x, p, t) \quad (15)$$

where $f_0(p)$ is the equilibrium distribution, whereas $\delta f(x, p, t)$ is small deviation from the equilibrium distribution of mass points. The equilibrium distribution is assumed to be of Maxwellian type, and for the sake of simplicity we assume non-relativistic scenario with the Hamiltonian given by eqn.(11a). The Maxwellian distribution of energy of the mass points is given by

$$f_0(p) = N_0 \exp \left[-\frac{\beta(x) \left(m_0 c^2 + \frac{p^2}{2m_0} \right)}{k_B T} \right] \quad (16)$$

where N_0 is a constant depends on the temperature of the system. Considering that the small perturbation is because of monochromatic type gravity waves, we can write

$$\delta f(x, p, t) \propto \exp[i(kx - \omega t)] \quad (17)$$

On substituting in Vlasov equation (eqn.(14)) the distribution function f , which is a sum of f_0 , the equilibrium part and δf , the small perturbation part, which are given by eqns.(16) and (17) respectively, we have

$$-i\omega \delta f + ikv\beta(x) \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{-1/2} \delta f = m_0 \alpha \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2} \frac{\partial f_0}{\partial p} \quad (18)$$

Hence

$$\delta f = \frac{m_0 \alpha \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2} \frac{\partial f_0}{\partial p}}{i \left[kv\beta(x) \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{-1/2} - \omega \right]} \quad (19)$$

where

$$\beta(x) = 1 + \frac{\alpha x}{c^2} \quad (20)$$

Since the Rindler space is locally flat, we may use the results of standard classical electrodynamics and can write the polarization equation for gravity in the following form

$$\frac{dP}{dx} = \delta\rho \quad (21)$$

where $\delta\rho$ is the small change in matter density caused by the incident gravity waves or in other words a small deviation of mass distribution, given by

$$\delta\rho = m_0 \int \delta f d^3p \quad (22)$$

Assuming that the polarization function of the mass deviation function P also varies harmonically and considering the relation from the classical electrodynamics

$$P = \frac{(\epsilon_l - 1)\alpha}{4\pi} \quad (23)$$

we have

$$\epsilon_l = 1 + \frac{4\pi m_0^2}{k} \int \frac{\left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2}}{\left[\frac{kv\beta(x)}{\left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2}} - \omega\right]} \frac{\partial f_0}{\partial p} d^3P \quad (24)$$

where ϵ_l may be assumed to be the gravitational permittivity or dielectric constant. Defining

$$z^2 = \frac{\beta(x)p^2}{2mk_B T} \quad \text{and} \quad \gamma = \frac{k_B T}{\beta(x)c^2} \quad (25)$$

we have

$$\epsilon_l = 1 + \frac{8\pi m_0^2 N}{k^2 (k_B T \beta(x))^{1/2}} \exp(-m_0/\gamma) \int_0^\infty \frac{(\gamma^2 z^5 + 2\gamma z^3 + z) \exp(-z^2)}{\gamma u z^2 - z + u} dz \quad (26)$$

where

$$u = \frac{\omega}{(k(2k_B N T \beta(x))^{1/2})} \quad (27)$$

This is the general expression for the gravitational dielectric constant as a function of frequency and wave number of the incident gravity wave. Now instead of going into all possible harmonics of the incident gravity waves, for the sake of mathematical convenience, we consider two extreme regions. The high frequency range, when $u \gg 1$, we have

$$\epsilon_l = 1 + A \int_0^\infty \frac{(\gamma^2 z^5 + 2\gamma z^3 + z) \exp(-z^2)}{\gamma u z^2 - z + u} dz \quad (28)$$

where

$$A = -\frac{8\pi N m_0^2}{k^2 (2k_B \beta(x))^{1/2}} \exp(-m_0/\gamma) \quad (29)$$

On integrating over z , we get

$$\epsilon_l = 1 + 2\pi i A \left[\frac{(1 - 3\gamma u^2 \gamma^2 u^4)}{\gamma^2 u^4} + \frac{2(1 - \gamma u^2)}{\gamma u^2} + 1 \right] \exp\left(-\frac{1 - 2\gamma u^2 + (1 - 4\gamma u^2)^{1/2}}{2\gamma^2 u^2}\right) \quad (30)$$

Now in the high frequency range, it is quite possible that the quantity within the square root in the exponential term may become negative. Then the permittivity may be written as

$$\epsilon_l = \epsilon_l^{(R)} + \epsilon_l^{(I)} \quad (31)$$

where $\epsilon_l^{(R)}$ and $\epsilon_l^{(I)}$ are the real and imaginary components of the permittivity ϵ_l in the high frequency range and are given by

$$\epsilon_l^{(R)} = 1 + 2\pi A \left[\frac{(1 - 3\gamma u^2 \gamma^2 u^4)}{\gamma^2 u^4} + \frac{2(1 - \gamma u^2)}{\gamma u^2} + 1 \right] \exp \left(-\frac{1 - 2\gamma u^2}{2\gamma^2 u^2} \right) \sin \left[\frac{(4\gamma u^2 - 1)^{1/2}}{2\gamma^2 u^2} \right] \quad (32)$$

and

$$\epsilon_l^{(I)} = 1 + 2\pi A \left[\frac{(1 - 3\gamma u^2 \gamma^2 u^4)}{\gamma^2 u^4} + \frac{2(1 - \gamma u^2)}{\gamma u^2} + 1 \right] \exp \left(-\frac{1 - 2\gamma u^2}{2\gamma^2 u^2} \right) \cos \left[\frac{(4\gamma u^2 - 1)^{1/2}}{2\gamma^2 u^2} \right] \quad (33)$$

respectively

In fig.(1) we have shown the variation of the ratio of real part of ϵ_l and A with the frequency of the radiation. It has been observed from this figure that the real part becomes exactly one for the the large values of frequency. Since A is independent of frequency, we may conclude that the gravitational dielectric constant of the medium for gravity waves in this region is $\propto A$. In fig.(2) we have shown the frequency dependence of the corresponding imaginary part

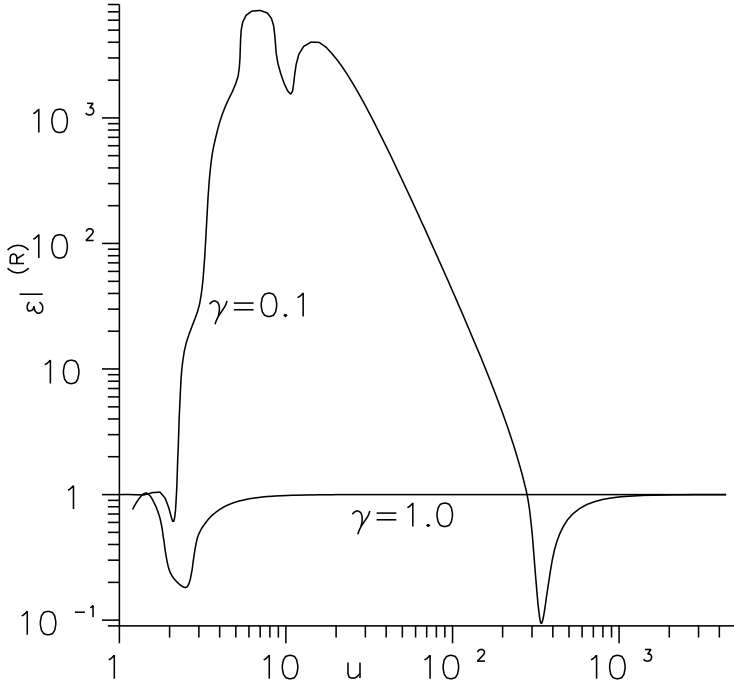


FIG. 1: The variation of the scaled real part of ϵ_l with frequency in the high frequency region

of permittivity. It is evident from this figure that the imaginary part becomes extremely small for the high value of radiation frequency. Further, the actual value of the gravitational permittivity becomes exactly one, a real quantity for high values of α . This is because as gravitational field $\alpha \rightarrow \infty$, the quantity $\gamma \rightarrow 0$, then the exponential term present in the expression for permittivity makes it exactly equal to one. Therefore we expect that the region very close to the event horizon of a black hole, where the gravitational field is extremely high, is transparent for the propagation of gravity waves. Therefore, there will be no gravitational bending of plane gravity waves near strongly gravitating objects. On the other hand, it is well known that there are gravitational bending of electromagnetic waves while pass through strong gravitational field. Further, it has been shown that free space behaves like a refracting medium in presence of gravity [1, 11]. The Rindler space, which is locally flat also behaves like a refracting medium. Further, it has been observed that since strong gravitational field makes the refracting index quite high, the speed of light decreases with the increase in gravitational field. Of course the gravitational refractive index for optical waves remains always real. On the other hand for gravity waves there is no such effect of strong gravitational field. The

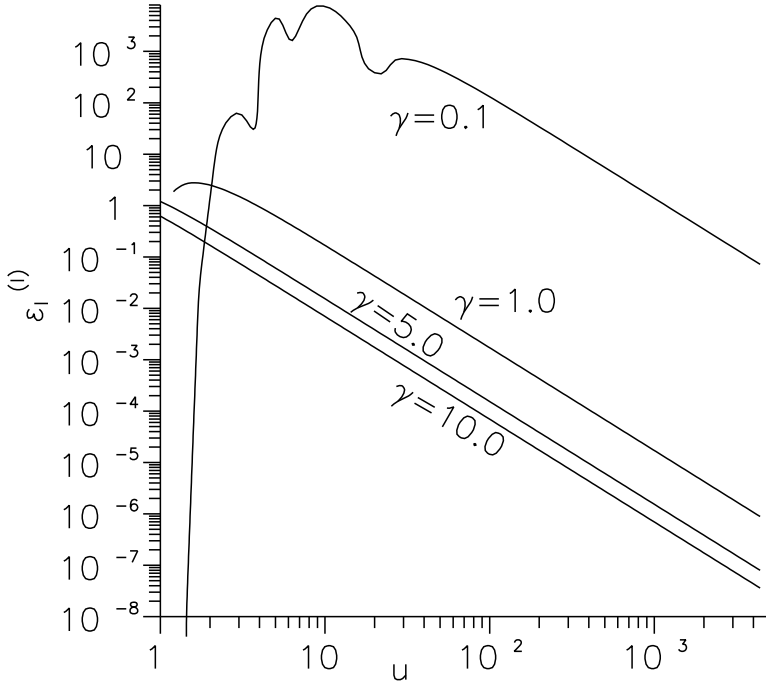


FIG. 2: The variation of the scaled imaginary part of ϵ_l with frequency in the high frequency region

physical reason behind such non-interacting nature of gravity waves and strong gravitational field is as follows: It is well known that the propagation of gravity waves is the fluctuation of space-time coordinates at different points from their flat configuration. It is also well established that the distortion of space-time geometry near a strongly gravitating object is quite high and since in this case during the propagation of gravity wave, where the space time fluctuation effect is negligibly small compared to such strong distortion of space-time geometry, we may conclude that there is almost no interaction of gravity waves with the gravitational field of strongly gravitating objects.

We next consider the low frequency gravity waves, i.e., $u \ll 1$. In this case the permittivity becomes

$$\epsilon_l = \epsilon_l^{(R)} + i\epsilon_l^{(I)} \quad (34)$$

where the real part is given by

$$\epsilon_l^{(R)} = 1 \quad (35)$$

and is independent of frequency. Therefore it is always unity. The corresponding imaginary part is given by

$$\begin{aligned} \epsilon_l^{(I)} = & 2\pi A[-2\gamma^2 u^6 + \gamma^2 u^5 + (5\gamma^2 - 14\gamma)u^4 + 2\gamma u^3 + (16\gamma - 34)u^2 \\ & + u + \left(\frac{4}{\gamma} - \frac{14}{\gamma^2}\right)\frac{1}{u^2} + \left(\frac{1}{\gamma^2} - \frac{2}{\gamma^3}\right)\frac{1}{u^4} - \frac{34}{\gamma} + 17] \exp(-1/\gamma^2 u^2) \end{aligned} \quad (36)$$

Since the real part of the permittivity in low frequency region is always one, we have plotted the variation of imaginary part with frequency in fig.(3) for various values of γ . Here also we have actually plotted the ratio of the imaginary part of permittivity and A . In this case the imaginary part saturates to the value ~ 100 for relatively high frequency range. Since the imaginary part is quite large, we expect that there will be reasonable amount of absorption of low frequency radiation in the medium, i.e., in the plasma of point galaxies and the low frequency part of gravity waves may not be possible to observe.

3. BOLTZMANN EQUATION FOR STELLAR PLASMA IN RINDLER SPACE

We next consider a collisional stellar plasma in Rindler space. We put the collision term by hand on the right hand side of the Vlasov equation and is given by

$$\frac{\partial f}{\partial t} - v\beta(x) \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{-1/2} \frac{\partial f}{\partial x} - m_0 \alpha \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2} \frac{\partial f}{\partial p} = C[f] \quad (37)$$

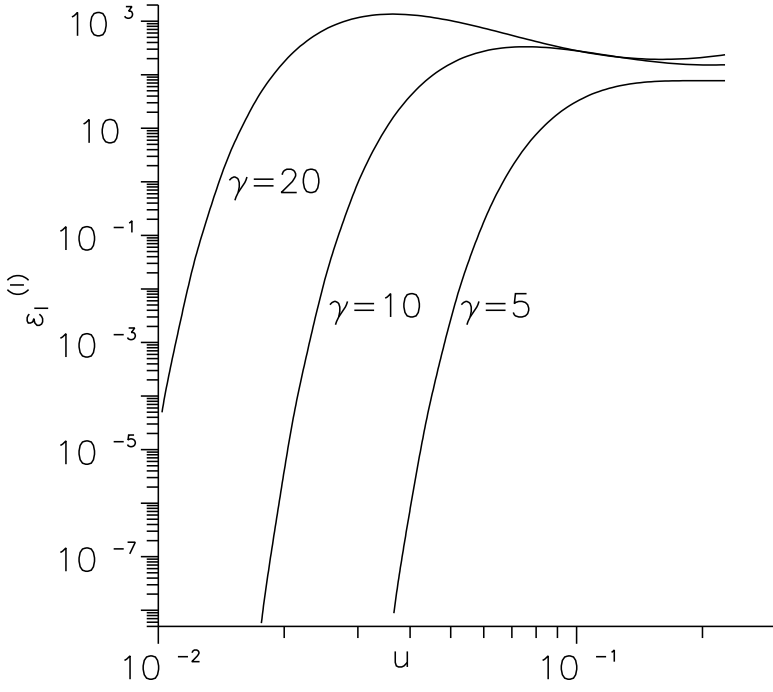


FIG. 3: The variation of the scaled imaginary part of ϵ_l with frequency in the low frequency region

This is the Boltzmann equation in Rindler space. Since it is not possible to evaluate the collision term from the cross section of the elementary processes, we make relaxation time approximation, given by

$$C[f] = -\frac{f(x, p, t) - f_0(p)}{\tau} \quad (38)$$

where τ is the relaxation time and it is assumed that the system is very close to the equilibrium configuration. The function $f_0(p)$ is the equilibrium distribution function. For the evaluation of kinetic coefficients, for the sake of simplicity we assume that $f_0(p)$ is Maxwellian in nature and in the present scenario the equilibrium distribution is given by

$$f_0(p) = C \exp\left(\frac{\mu - \varepsilon(x, p) + Vp}{k_B T}\right) \quad (39)$$

where $\mu \equiv \mu(x, t)$, the local chemical potential, $T \equiv T(x, t)$, the local temperature and $V \equiv V(x, t)$, the local flow velocity, which are also changing with time and the single particle energy in the non-relativistic approximation is given by

$$\varepsilon(x, p) = \beta(x) \left(\frac{p^2}{2m_0} + m_0 c^2 \right) \quad (40)$$

Here our intention is to obtain the kinetic coefficients of the stellar plasma in Rindler space. In doing so we proceed as follows: Now keeping only the equilibrium distribution function on the left hand side of Boltzmann equation, we have

$$\left[\frac{\partial}{\partial t} + v\beta(x) \frac{\partial}{\partial x} - m_0 \alpha \left(1 + \frac{p^2}{2m_0^2 c^2} \right) \frac{\partial}{\partial p} \right] f_0(p) = C[f] \quad (41)$$

where $C[f] = -\delta f(x, p, t)/\tau$ in linear approximation, where $\delta f(x, p, t)$ is the small deviation of distribution function from the equilibrium configuration. Then following Huang [13] for the usual version of Boltzmann equation and taking into account

$$\frac{\partial f_0}{\partial \varepsilon} = -\frac{f_0}{k_B T} \quad (42)$$

we have

$$\begin{aligned} & \left[\frac{(\varepsilon - h)\beta(x)}{T} v \frac{\partial T}{\partial x} + m_0 \beta(x) u_x u_y \zeta_{x,y} + m_0 \beta(x) \left(1 + \frac{p^2}{2m_0^2 c^2} \right) \alpha v \right. \\ & \left. + m_0 (1 - \beta(x)) v \frac{\partial v}{\partial t} - \frac{\varepsilon - h + T C_p}{C_v / k_B} \frac{\partial u_x}{\partial x} \right] \frac{f_0}{k_B T} = - \frac{\delta f}{\tau} \end{aligned} \quad (43)$$

In the left hand side there are all possible driving forces, e.g., thermal conduction, viscous flow (both shear and bulk) and flow under gravity. Here

$$\zeta_{xy} = \frac{1}{2} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \quad (44)$$

is the shear tensor. The term with the driving force $\partial u_x / \partial x$ represents the bulk viscosity. In our present study we discard this term. Our next task is to obtain the three different types of transport coefficients, viz, thermal conductivity, shear viscosity coefficient and coefficient of gravitational flow.

4. SHEAR VISCOSITY COEFFICIENT

To obtain the shear viscosity coefficient we consider only the driving force associated with shear flow. The relevant form of the deviation of distribution function from its equilibrium structure is given by

$$\delta f = - \frac{\tau}{k_B T} \frac{m_0 \beta(x)}{\left(1 + \frac{p^2}{m_0^2 c^2} \right)} u_x u_y \zeta_{xy} \quad (45)$$

Now from the definition of viscous flow we can write down the relevant form for the expression of pressure tensor in the following form

$$\Pi_{xy} = n \int d^3 p p_x v_z (f_0(p) + \delta f(x, p, t)) \quad (46)$$

where n is the number density and the above equation basically indicates the momentum transfer along z -direction, while the actual flow is along x -direction. Now it is a matter of simple algebra to show that in the expression for pressure tensor, the equilibrium part does not contribute. This is also evident from the physical nature of equilibrium configuration. Therefore after integrating over momentum in the rest frame of the fluid element when $u \rightarrow v$ for all the components and comparing with equation for viscous flow, given by

$$\Pi_{xy} = -\eta \frac{du_x}{dz} \quad (47)$$

we have for the coefficient of shear viscosity

$$\eta = \frac{n \tau k_B T}{\beta^4(x)} \exp \left(- \frac{2 m_0 c^2 \beta(x)}{k_B T} \right) \quad (48)$$

It is obvious that for $\alpha = 0$, i.e., in absence of gravitational field when $\beta = 1$, we get back the conventional result in absence of external gravitational field, or in flat space-time. Whereas for ultra strong gravitational field, i.e., for $\alpha \rightarrow \infty$ or $\beta \rightarrow \infty$ the coefficient of viscosity becomes vanishingly small.

5. THERMAL CONDUCTIVITY

In this case the thermal current is given by

$$j_\epsilon^x = \int d^3 p \varepsilon v_x f(x, p, t) \quad (49)$$

Again the equilibrium part does not contribute. Therefore we have

$$j_\epsilon^x = \int d^3 p \varepsilon v_x \delta f(x, p, t) \quad (50)$$

Further the relevant form of the deviation part of distribution function is given by

$$\delta f(x, p, t) = \frac{\tau f_0(p)}{k_B T} \frac{(\epsilon - h)\beta(x)}{T} u \frac{dT}{dx} \quad (51)$$

Substituting this value in the thermal current and considering the standard definition of thermal current

$$j_\epsilon^x = -K \frac{dT}{dx} \quad (52)$$

we have after integrating over momentum in the local rest frame

$$K = \frac{n\tau}{4m_0 T \beta^{3/2}(x)} \exp\left(-\frac{m_0 c^2 \beta(x)}{k_B T}\right) \left[\frac{35k_B^2 T^2}{\beta(x)} + 10k_B T m_0 c^2 + \left(\frac{10k_B T}{\beta(x)} + m_0 c^2\right) (m_0 c^2 \beta(x) - \frac{5}{2} k_B T) \right] \quad (53)$$

Exactly like the shear viscosity coefficient, the thermal conductivity also reduces to its standard form for $\beta = 1$ and vanishes for large gravitational field.

6. COEFFICIENT OF GRAVITATIONAL FLOW

This is the first time such coefficient is obtained. We are the first to calculate this quantity. The relevant form of Boltzmann equation in this case is given by

$$\left[n_0(1 - \beta(x))v \frac{dv}{dt} + m_0 \beta(x) \left(1 + \frac{p^2}{2m_0^2 c^2}\right) \alpha v \right] \frac{f_0(p)}{k_B T} = -\frac{\delta f}{\tau} \quad (54)$$

From the equation of motion we have

$$m_0(1 - \beta(x))v \frac{dv}{dt} = -m_0(1 - \beta(x)) \left(1 + \frac{p^2}{2m_0 c^2}\right) \alpha v \quad (55)$$

Hence the deviation part of the distribution function may be written as

$$\delta f = \frac{(1 - 2\beta(x))m_0 \tau}{k_B T} \left(1 + \frac{p^2}{2m_0^2 c^2}\right) \alpha v \quad (56)$$

Now we may define the mass flow current under the influence of gravity in the following form

$$j_m(x) = m_0 \int d^3 p v_x f(x, p, t) \quad (57)$$

Assuming again $f(x, p, t) = f_0(p) + \delta f(x, p, t)$ and since $f_0(p)$ does not contribute, we have

$$j_m(x) = m_0 \int d^3 p v_x \delta f(x, p, t) \quad (58)$$

On substituting $\delta f(x, p, t)$ in the above expression, integrating over momentum in the local rest frame and finally from the analogy of charge current flow, we can write

$$j_m(x) = \sigma \alpha \quad (j = \sigma E \text{ in the electrical case}) \quad (59)$$

we have the coefficient for the gravitational flow of mass

$$\sigma = \frac{n\tau(1 - 2\beta(x))}{3c^2 \beta^{5/2}} \left(\frac{5k_B T}{2} + m_0 c^2\right) \exp\left(-\frac{m_0 c^2 \beta(x)}{k_B T}\right) \quad (60)$$

Since $(1 - 2\beta(x)) = -(1 + \frac{2\alpha x}{c^2})$, a negative quantity, the coefficient of mass flow under gravity is also negative. The physical reason for negative value is because unlike all other kind of flows, where current travels from high potential to low potential region, the mass flow occurs from low gravity region to high gravity region. Further like all other kinetic coefficients this particular kinetic coefficient also vanishes for high gravitational field. However, the formalism to obtain this kinetic coefficient is not valid for zero gravitational field. Therefore the stellar matter in presence of ultra strong gravitational field behaves like an ideal fluid. Which may occur at the vicinity of event horizon. This may also be true if the matter is created near the event horizon of a black hole through Hawking radiation [14]. According to principle of equivalence the uniform acceleration of the frame equivalent to a constant gravitational field of a rest frame, then the radiation or material particles observed in quantum vacuum by the uniformly accelerated frame as Unruh effect [15] will also behave like an ideal fluid.

7. CONCLUSION

We have seen that in the high frequency region of gravity wave, the real part of gravitational permittivity converges to unity at relatively higher frequency values, whereas the imaginary part becomes extremely small. We have also observed that the actual value of the permittivity becomes exactly one, a real number for very high gravitational field, which makes the medium transparent for gravity waves. On the other hand in the low frequency region, the real part is always unity, whereas the imaginary part saturates to a value ~ 100 , which makes the medium strongly absorbing in that frequency zone.

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- [1] Landau L.D. and Lifshitz E.M., The Classical Theory of Fields, Butterworth-Heimenann, Oxford, (1975).
 - [2] W.G. Rosser, Contemporary Physics, **1**, 453, (1960).
 - [3] N.D. Birrell and P.C.W. Davies, Quantum Field Theory in Curved Space, Cambridge University Press, Cambridge, (1982).
 - [4] Torres del Castillo G.F. and Perez Sanchez C.L., Revista Mexican De Fisika 52, 70, (2006).
 - [5] M. Socolovsky, Annales de la Fondation Louis de Broglie **39**, 1, (2014).
 - [6] C.G. Huang and J.R. Sun, arXiv:gr-qc/0701078, (2007).
 - [7] Domingo J Louis-Martinez, Class. Quantum Grav., **28**, 036004, (2011).
 - [8] D. Percoco and V.M. Villaba, Class. Quantum Grav., **9**, 307, (1992).
 - [9] S. De, S. Ghosh and S. Chakrabarty, Astrophys and Space Sci, 360:8, DOI 10.1007/s10509-015-2520-3. (2015).
 - [10] S. De, S. Ghosh and S. Chakrabarty, Mod. Phys. Lett. A **30**, 1550182 (2015).
 - [11] Soma Mitra and Somenath Chakrabarty (to be submitted, 2017).
 - [12] Physical Kinetics, E.M. Lifshitz and L.P. Pitaevskii, Vol.10, Butterworth-Heimenann, Oxford, (1998).
 - [13] Statistical Mechanics, K. Huang, Wiley, NY, 1965.
 - [14] S.W. Hawking, Nature, **248**, 30, (1974); S.W. Hawking, Comm. Math. Phys. **43**, 199, (1975).
 - [15] W.G. Unruh, Phys. Rev. **D14**, 4, (1976); W.G. Unruh, Phys. Rev. **D14**, 870, (1976).